

# PERFORMANCE OF RF DOWNLINK WITH PERIODIC DATA FRAME<sup>1</sup>

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## ABSTRACT

Telemetry discrete spectrum components induced by the periodicity of an Attached Sync Marker may coincide with the RF carrier frequency causing unwanted interference while tracking the RF carrier. However, with the CCSDS recommended frame length of 10200, the power of all harmonics created by this periodicity are always 50 dB below the overall data power, hence there is no significant interference imposed on the carrier tracking. Additionally, it is found that insertion of the pseudorandomizer reduces the intensity of all harmonics caused by bursts of 1's and 0's, and therefore decreases any potential interference to the carrier tracking.

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<sup>1</sup> . The work described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration

## 1. Introduction

Telemetry discrete spectral components, due to the periodicity of the Attached Sync Marker (ASM) and burst of 1's or 0's, may coincide with the RF carrier, and thus cause unwanted interference. This paper investigates whether such an interference can arise in the telemetry downlink. Also spectral plots of the CCSDS pseudorandomizer [Ref 1., 2.] are presented.

### 1.1. Spectral Components due to Periodic Sync Word

According to the CCSDS telemetry format, each data frame is separated from the previous one by a 32 bit synchronization (sync) word written in hexadecimal form as 1 ACFFC 1D. The periodicity of the sync word creates discrete harmonics in the telemetry data spectrum. In this section, the possibility of such spectral components, causing interference to the RF carrier, is investigated.

We assume, for the sake of analysis, an all-zero frame with length  $N_f$  and a sync word with length  $N_s$ . Let  $A_m = \pm 1$  be the amplitude of the  $m$ th bit of the sync word,  $T_b$  the duration of data bit. The duration of a frame plus the sync word becomes  $T_0 = (N_f + N_s)T_b$ . The baseband telemetry periodic data waveform  $y(t) = \text{sync}(t) + \text{frame}(t)$  can be expanded into Fourier series as :

$$y(t) = \sum_{n=-\infty}^{\infty} Y_n e^{j\omega_n t}, \quad \omega_n = \frac{2\pi n}{(N_s + N_f) T_b} \quad (1)$$

where the spectral components  $Y_n$  can be computed from the Fourier transform as

$$Y_n = \frac{1}{T_0} \int_0^{T_0} y(t) e^{-jn\omega_0 t} dt = \frac{1}{T_0} \sum_{m=0}^{N_s-1} \int_m^{(m+1)T_b} A_m e^{jn\omega_0 t} dt + \int_{N_s T_b}^{(N_s+N_f)T_b} e^{-jn\omega_0 t} dt \quad (2)$$

Carrying out the integration yields the following expression for the spectral component  $Y_n$

$$Y_n = \frac{1}{N_s + N_f} \text{Sinc}\left(\pi \frac{n}{N_s + N_f}\right) \sum_{m=0}^{N_s-1} A_m e^{-j\pi n \frac{2m+1}{N_s + N_f}} - \frac{N_f}{N_s + N_f} \text{Sinc}\left(\pi n \frac{N_f}{N_s + N_f}\right) e^{-j\pi n \frac{2N_s + N_f}{N_s + N_f}} \quad (3)$$

Spectral component  $Y_0$  for  $N_s = 32$  becomes :

$$Y_0 = \frac{1}{N_s + N_f} \sum_{m=0}^{N_s-1} A_m + \frac{N_f}{N_s + N_f} = \frac{4}{32} \frac{9}{19+13} + \frac{N_f}{32+N_f} = \frac{6+N_f}{32+N_f} \quad (4)$$

The power spectral density of the baseband waveform  $y(t)$  contains discrete components at  $\pm n\omega_0$

$$S_y(f) = \sum_{n=-\infty}^{\infty} |Y_n|^2 \delta(f - nf_0) \quad (5)$$

In Fig.1, plots of (5) up to  $n=100$ , for  $N_f = 10200, 1020, 102$  demonstrate the dramatic increase in the power of the sync word harmonics, as the frame length becomes shorter. However, for the CCSDS recommended frame length of 10200, the power of all harmonics other than the fundamental is always 50 dB below the overall data power.

If  $y(t)$  modulates a square wave subcarrier / carrier, then the power spectral density of the resulting PCM/PSK/PM signal centered around the carrier is the following :

$$S_M(f) = S_M^+(f-f_c) + S_M^-(f+f_c); \quad \text{where:}$$

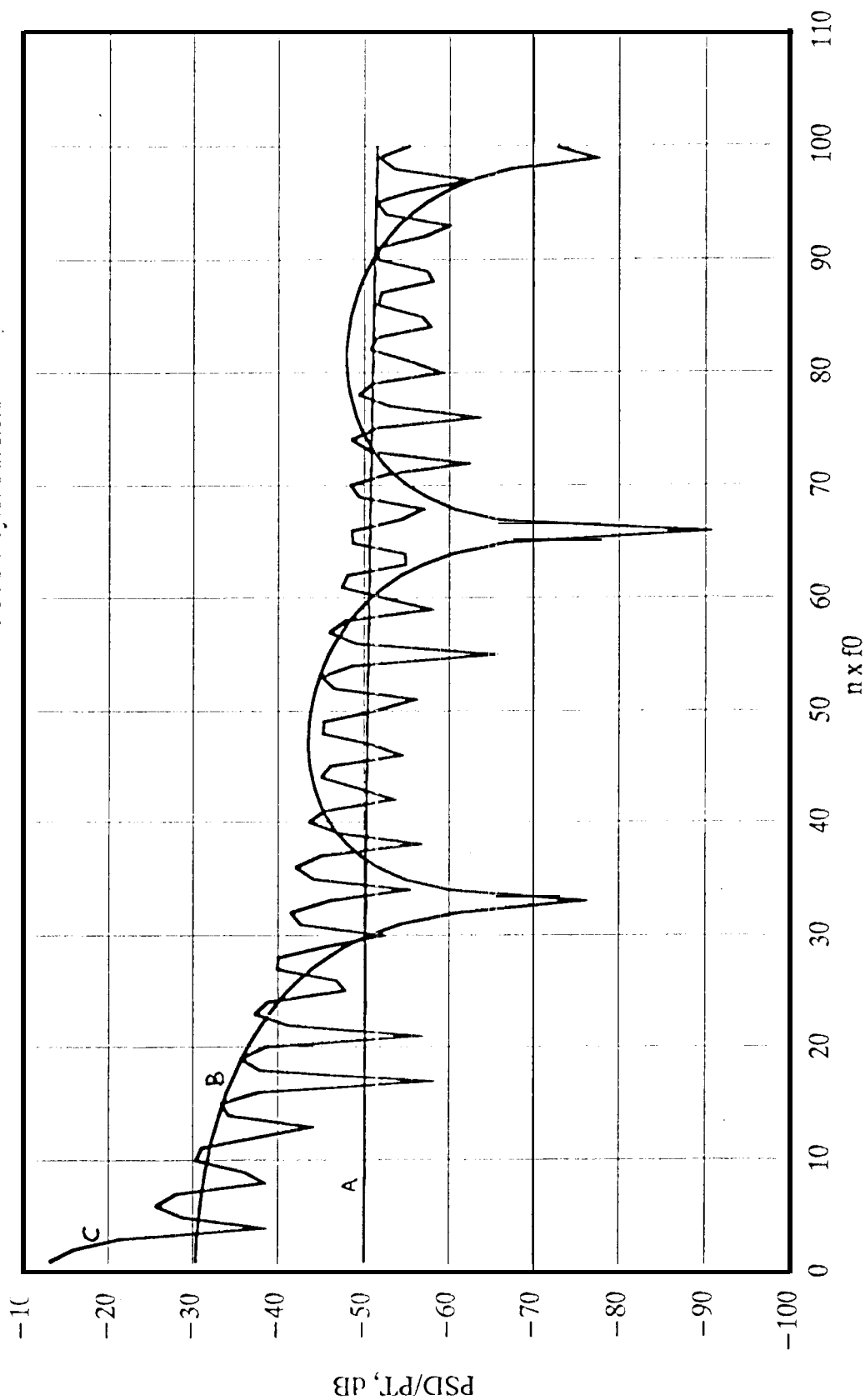
$$S_M^+(f-f_c) = \frac{2}{\pi^2} \sum_{k=1}^{\infty} \left[ \frac{S_y(f-f_c - (2k-1)f_{sc}) + S_y(f-f_c + (2k-1)f_{sc})}{(2k-1)^2} \right], \quad (6)$$

$$S_M^-(f+f_c) = \frac{2}{\pi^2} \sum_{k=1}^{\infty} \left[ \frac{S_y(f+f_c - (2k-1)f_{sc}) + S_y(f+f_c + (2k-1)f_{sc})}{(2k-1)^2} \right]$$

For computational simplicity we choose to perform our analysis around the positive carrier frequency where the modulation spectrum is  $S_M^+(v)$ ,  $v = f - f_c$ :  
Substituting expression (5) for  $S_y(f)$  in (7), yields an expression for the discrete power spectrum of the PCM/PSK/PM signal :

**Fig.1 : Power Spectral Density of Baseband Telemetry**

All Zero TLM Frame + CCSDS Sync Marker



A  $N_f = 10200$  B  $1020$  C  $102$

$N_s = 32$  bit at Sync Marker  
 $f_0 = 1/T_0 = 1/(N_s + N_f)T_b$

$$S_M^*(\nu) \equiv \frac{2}{\pi^2} \left[ \sum_{k=1}^{\infty} \frac{S_y(\nu - (2k-1)f_{sc}) + S_y(\nu + (2k-1)f_{sc})}{(2k-1)^2} \right] \quad (7)$$

$$S_M^*(\nu) \equiv \frac{2}{\pi^2} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} \sum_{n=-\infty}^{\infty} |Y_n|^2 [\delta(\nu - nf_0 - (2k-1)f_{sc}) + \delta(\nu - nf_0 + (2k-1)f_{sc})] \quad (8)$$

Discrete harmonics of eg. (8) coincide (and therefore interfere) with the carrier if the following Diophantine equation has at least one solution :

$$nf_0 - (2k-1)f_{sc} = 0, \quad k=1,2,3,\dots, \text{ and } n=1,2,3,\dots \quad (9)$$

Using the definition for  $f_0$  from (1) and setting  $x_b \equiv f_{sc}/R_b$ , ( $R_b = 1/T_b$ ), the above equation becomes :

$$n - (2k-1)(N_s + N_p)x_b = 0, \quad n, k = 1, 2, 3, \dots \quad (10)$$

Equation (10) has an infinite number of discrete solutions if the subcarrier-frequency-to-bit-rate ratio  $x_b$  is an integer. As shown in the solutions presented in Table 1, below the interfering, harmonic power on the carrier is negligible compared to the overall data power in the link.

Table 1 : Solutions of (10) with $k=1$				
$x_b$	2			1.875
$N_f$	10200	1020	102	10200
$n$	20464	2104	268	19185
$ Y_n ^2$ , dB	-149.91	-155.74	-144.99	-152.36

### 11.1. CCSDS Pseudo-Randomizer

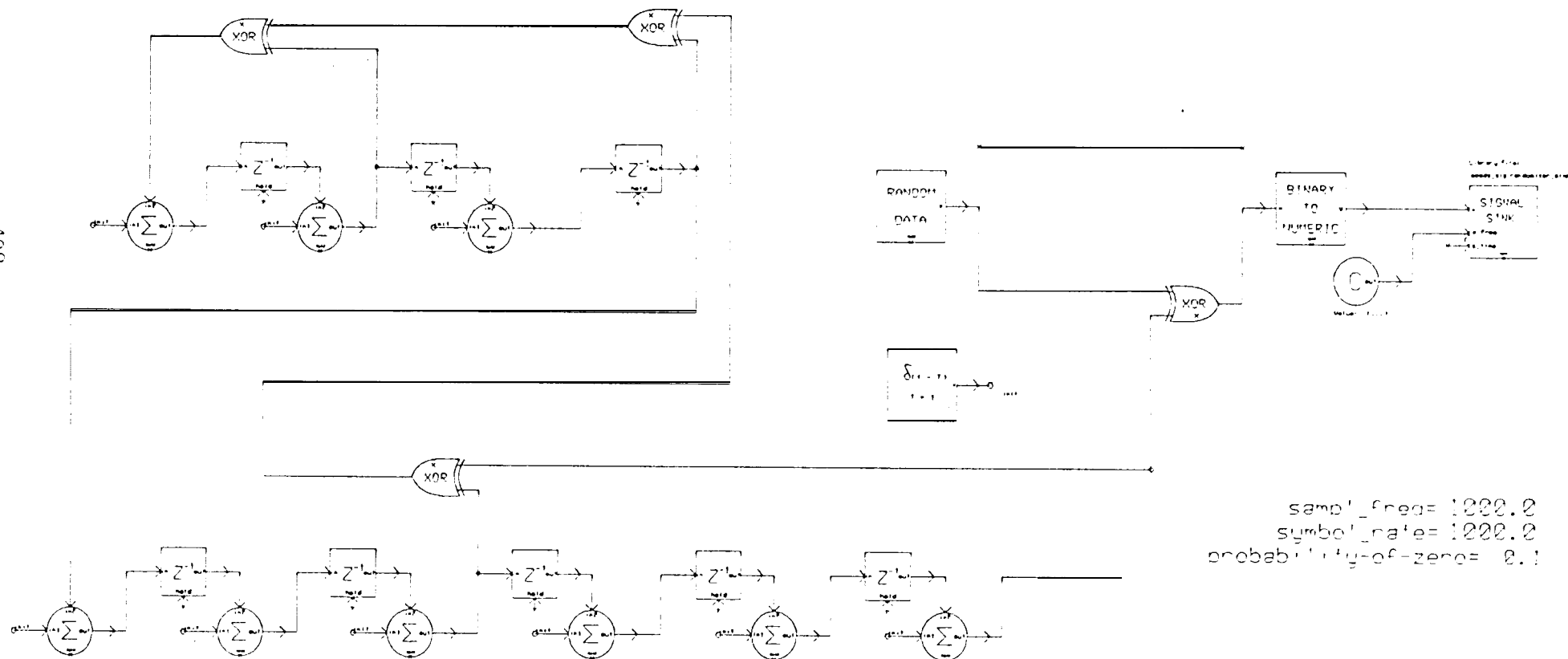
In order to avoid inadvertent creation of telemetry harmonics falling on the carrier frequency, CCSDS recommends "XORing" a pseudorandom sequence of length=255 bits with the transfer frame. The pseudorandom sequence is generated using the following polynomial :

$$h(x) = x^8 + x^7 + x^5 + x^3 + 1$$

Since there is such a repetitive pattern inside a transfer frame, there exist a possibility of undesirable harmonics generation. However, simulation of a random number generator with

Fig. 2 : CCSDS Pseudo-Randomizer

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probability of zero = 0.1 in the Comdisco SPW<sup>TM</sup> (See Fig. 2) demonstrated that inserting the pseudorandomizer keeps all harmonics 40 dB below total signal power.

In Fig. 3 the histogram and the spectrum of the random number generator is plotted with prob-of-zero = 0.1. A DC component is 5 db (or less ) below the total signal power as expected.

In Fig. 4 we see that by inserting the randomizer the DC component has been dispersed and all harmonics lie 40 db below the total signal power.

### **111. Conclusions**

Repetitive patterns such as Sync Markers do not generate significant interference to the carrier. However bursts of 1's or 0's generate strong components that may fall on the carrier frequency, therefore insertion of pseudorandomization in the transfer frame has been appropriately recommended by CCSDS.

### **IV. References**

1. CCSDS Blue Book : "TELEMETRY CHANNEL CODING", CCSDS 101.0-B-3, May 1982.
2. CCSDS Green Book : "TELEMETRY - Summary of Concept and Rationale", CCSDS 100.0-G-1, December 1987.

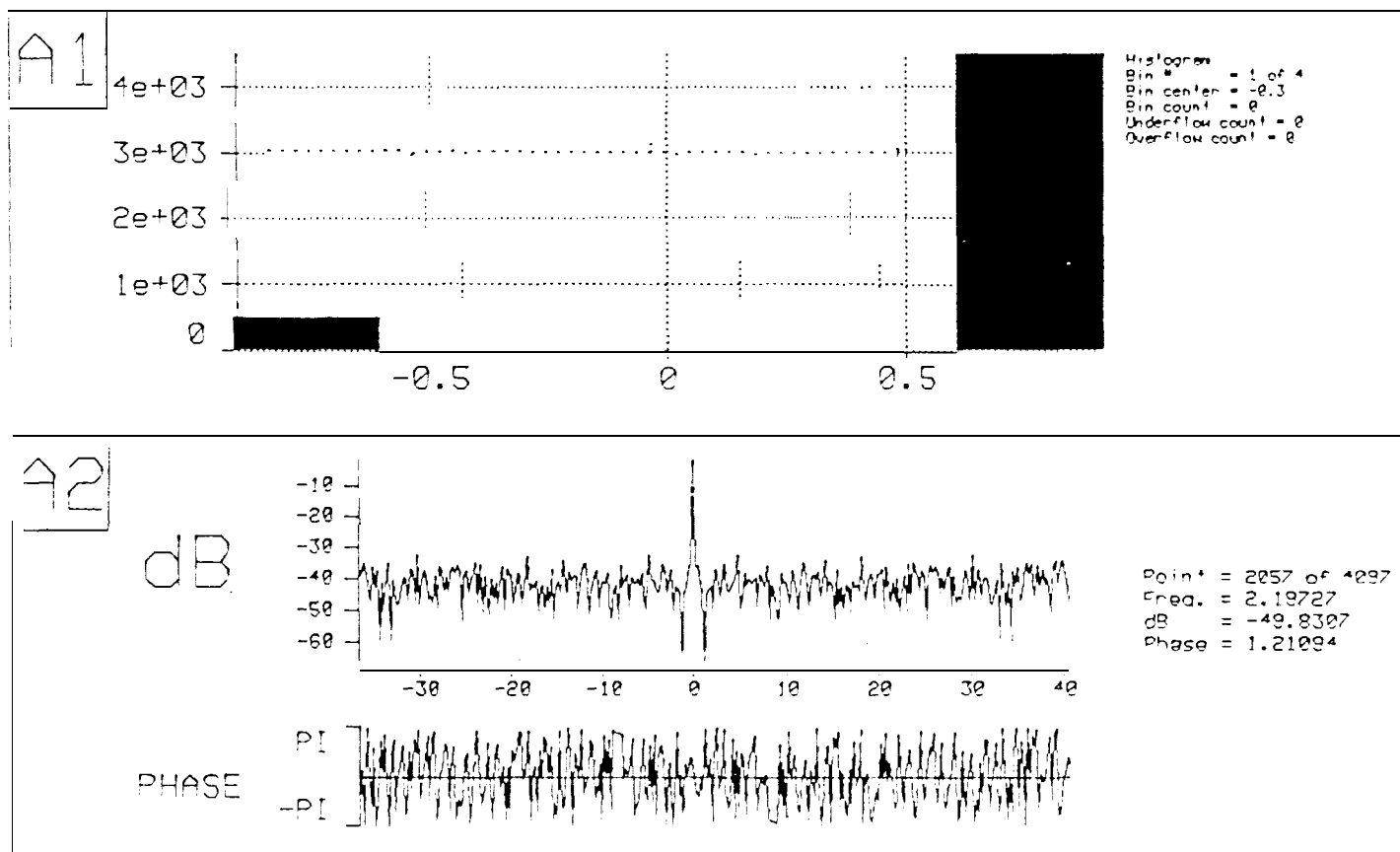


FIG. 3 A 1: Histogram of I's versus O's for a random number generator with prob-of-zero = 0.1.

A2 : FFT of a 4092 bit stream, showing a strong DC component.

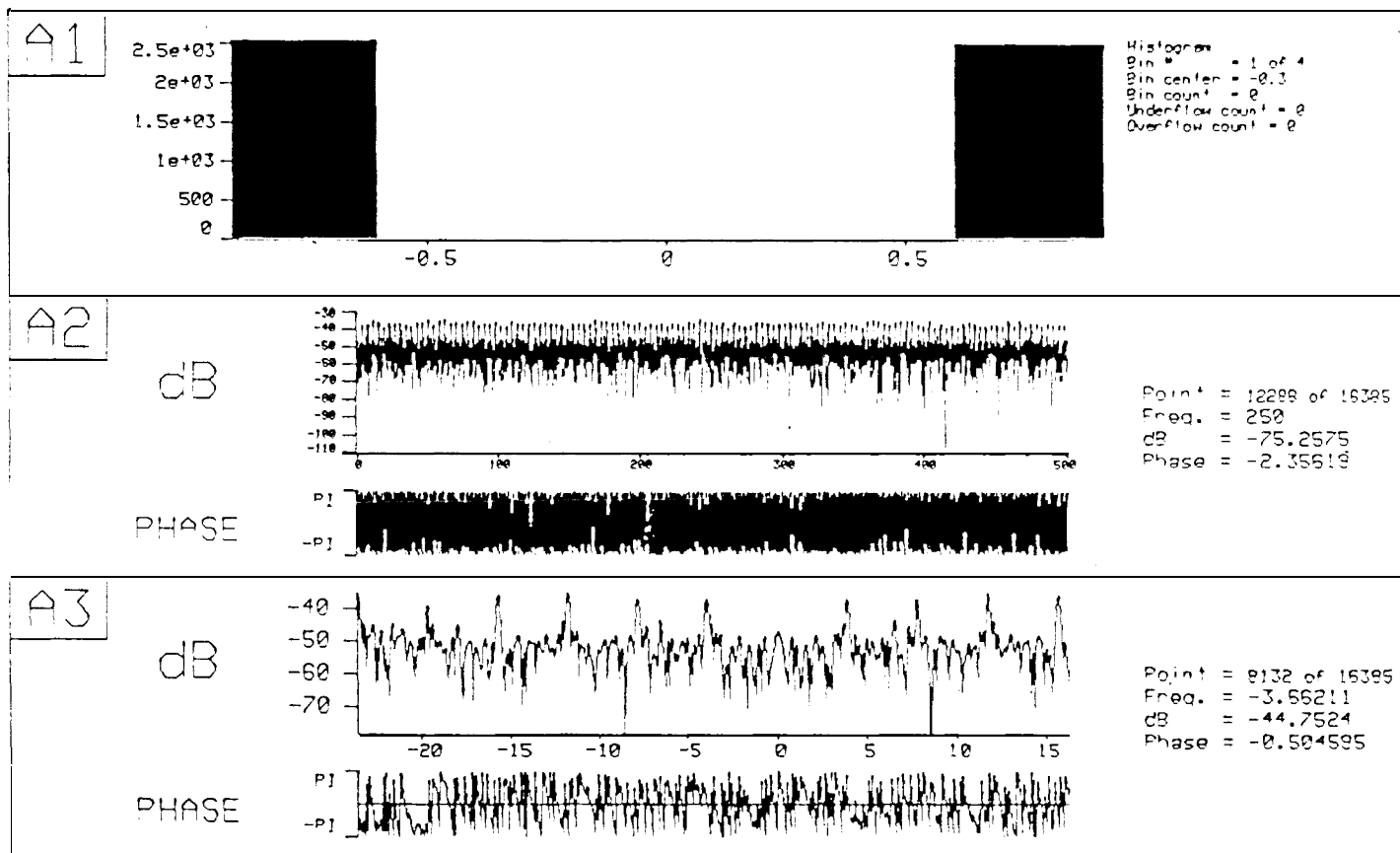


FIG. 4 A1: Histogram of 1's and 0's after the insertion of the pseudorandomizer. An almost 50% zero - 50% one situation.

A2: FFT of 16384 bits. All harmonics are approximately 40 dB or below the total signal power.

A3: FFT detail around the DC component. No significant DC component exists.